The Chain Rule

Lecture 44
Section 7.2

Robb T. Koether

Hampden-Sydney College

Mon, Apr 17, 2017

Reminder

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- Test #4 is this Friday, April 21.

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- Be there.

Objectives

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- Define the Chain Rule for partial derivatives.
- Practice using it.
- Learn how to use it to make approximations.

The Chain Rule

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- Let $f(x, y)$ be a function of x and y.

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- Suppose that x and y are both functions of another variable t.

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- Let $f(x, y)$ be a function of x and y.
- Suppose that x and y are both functions of another variable t.
- Then

$$\frac{df}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}.$$

Example

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Let $f(x, y) = x^2 - 3xy$, $x(t) = 2t + 1$ and $y(t) = t + 3$.

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(a) Express f as a function of t only. Then differentiate f to find $\frac{df}{dt}$.

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Let $f(x, y) = x^2 - 3xy$, $x(t) = 2t + 1$ and $y(t) = t + 3$.

(a) Express f as a function of t only. Then differentiate f to find $\frac{df}{dt}$.

(b) Find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{dx}{dt}$, and $\frac{dy}{dt}$. Then use the Chain Rule to find $\frac{df}{dt}$.

Example

Exercise 7.2.62 (modified)

A car dealer determines that if gasoline-electric hybrid automobiles are sold for x dollars apiece and the price of gasoline is y cents per gallon, then approximately H hybrid cars will be sold each year, where

$$H(x, y) = 2400 - 15x^{3/2} + 8(0.5y + 10)^{1/2}.$$

She estimates that t years from now, the hybrid cars will be selling for

$$x(t) = 40,000 + 500t$$

dollars apiece and that gasoline will cost

$$y(t) = 200 + 10(4t)^{1/2}$$

cents per gallon.

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(a) At what rate will the annual demand for hybrid cars be changing (with respect to time) 4 years from now?

Approximation Formula

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Based on the facts that

$$\Delta f \approx f'(t)\Delta t, \quad \Delta x \approx x'(t)\Delta t, \quad \text{and} \quad \Delta y \approx y'(t)\Delta t,$$

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it follows that

$$\Delta f \approx \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y.$$

Example

Example 7.2.62 (modified and extended)

Continuing Exercise 7.2.62, use derivatives to estimate the annual change in hybrid-car sales *during* the fifth year from now.